

## FIR window method: A comparative Analysis

Gopika P

Assistant Professor, Department of Electronics, S.E.A.S, Rai Technology University, Bangalore, India.

---

**Abstract :** This paper includes a concise record of data windows along with their significant performance parameters from which the different windows can be compared. Windows impact on many attributes of a signal processor which includes detectability , resolution, dynamic range, confidence and ease of implementation. Various Computer Aided Design tools are available for this purpose. The Filter Design and Analysis tool available with MATLAB provides a method to design. In a signal when we compute the frequency content, usually we tend to pick a limited-duration snapshot which results in errors because the signal actually lasts for a longer time. Windowing is a way to reduce these errors, though it cannot eliminate them completely. The problem identified in this paper is to allow performance comparisons between different FIR windows; mainly Hamming, Hann and Blackman window; based on some major parameters.

**Keywords -** FIR windows , MATLAB Signal processing , Window function

---

### I. Introduction

Windowing method uses a function called window function. It is also called tapering function. A window function is a mathematical function that is zero-valued outside of some chosen interval. When another function is multiplied by some window function , the product is also zero-valued outside the interval. In signal processing a filter removes unwanted component from the signal. The filters can be classified as lowpass, highpass, bandpass and bandstop based on their frequency characteristics. Window method is the most conventional method used for designing FIR filters. During frequency analysis we assume that the snapshot repeats. It is often found that the ends of the snapshots also does not smoothly blend. We can suppress the discontinuities and the resulting spurious high frequencies in the frequency analysis , by tapering the signal to zero at the start and end of the recording period. This is the basis of windowing[1][2]. Different filters are used based on the end effect they make on the input signal so that they alter the signal in some useful way. In the present day signal processing finds its application in telecommunication, military, space research , image processing , pattern recognition and in all these areas filters are an unavoidable electronic device. Digital filters are having obvious advantage over the olden days filters designed with passive components. In addition the digital filters provide more flexibility as their design can be changed easily using software[3].

Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters are among the two fundamental types of digital filters. The classification of the filters indicates that impulse response of FIR filter is finite. But an IIR filter has an impulse response of infinite duration. While implementing an FIR filter needs no feedback. It is not a recursive filter. For this reason FIR filters are much modest than IIR filters. Among the various methods used to implement an FIR filter window methods are widely used. This paper presents a comparative study of the commonly used window techniques.

### II. Problem Formulation

The initial step towards filtering technique is to identify the filter coefficients. Digital filter design involves usually the following basic steps. Determine a desired or a set of desired responses. Select a class of filters for approximating the desired responses. Establish a criterion of 'goodness' for the responses of a filter in the selected class compared to the desired response. Develop a method for finding the best member in the filter class. Analyse the filter performance.

### III. Finite Impulse Response Filter

The generalized form of an N-tap FIR filter is

$$Y(n) = x(n)*h(k) \\ = \sum_{k=0}^{N-1} h(k)x(n-k)$$

Where h(k) is the filter coefficient array and x(n-k) is the input data array to the filter. The number of taps relates to the filter performance. An N-tap filter requires N multiply-accumulate cycles. The output signal is determined as

$$Y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) + \dots\dots\dots + x(n)h(0)$$

The transfer function of a causal FIR filter is obtained by taking the z-transform of impulse response of FIR filter  $h(n)$ [4].

$$H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n}$$

$$= h(0) + h(1)Z^{-1} + h(2)Z^{-2} + h(3)Z^{-3} + \dots + h(N-1)Z^{-(N-1)}$$

FIR filters can be designed in different ways. There are many straightforward techniques for designing FIR digital filters to meet arbitrary frequency and phase response specifications, such as window design method or frequency sampling techniques. The Window method is the most popular and effective method because this method is simple, convenient, fast and easy to understand. The main advantage of this design technique is that the impulse response coefficient can be obtained in closed form without the need for solving complex optimization problems.

There are essentially three well-known methods for FIR filter design namely:

- (a) The window method
- (b) The frequency sampling technique
- (c) Optimal filter design methods

The window method is one of the simplest methods of designing FIR digital filters. It is well suited for designing filters with simple frequency response shapes, such as ideal lowpass filters. Mostly used Fixed window functions are; Rectangular window, Hann window, Hamming window and Blackman window. On the other hand the Kaiser window is a kind of adjustable window function. The most commonly used windows are discussed in this paper. The most important parameters in window weighting in FFT analysis are highest side-lobe level and the worst case processing loss. In both cases the lower the better[5].

### 3.1. HANN Window

This is also known as a cosine taper. It starts at 0, rises to 1 in the middle of the period, and then goes smoothly back down to zero at the end.

The window function of a causal Hann window is given by,

$$\omega(n) = \begin{cases} 0.5 - 0.5\cos\frac{2\pi n}{N-1}, & 0 < n < N-1 \\ 0, & \text{otherwise} \end{cases}$$

The window function of a non-causal Hann window is given by,

$$\omega(n) = \begin{cases} 0.5 + 0.5\cos\frac{2\pi n}{N-1}, & 0 < n < N-1 \\ 0, & \text{otherwise} \end{cases}$$

While the Hann window does a good job of forcing the ends to zero, it also adds distortion to the wave form being analyzed in the form of amplitude modulation; i.e., the variation in amplitude of the signal over the time record. The Hann window should always be used with continuous signals, but must never be used with transients. The reason is that the window shape will distort the shape of the transient, and the frequency and phase content of a transient is intimately connected with its shape. The measured level will also be greatly distorted. Even if the transient were in the center of the Hann window, the measured level would be twice as great as the actual level because of the amplitude correction the analyzer applies when using the Hann weighting.

A Hann window is used to design Lowpass filter, Highpass filter, Bandpass and Bandstop filter. Since speech signal can be considered for filtering, we use the sampling frequency here as 8000Hz. The simulation of filters are done using FDA tool of Signal Processing toolbox in MATLAB.

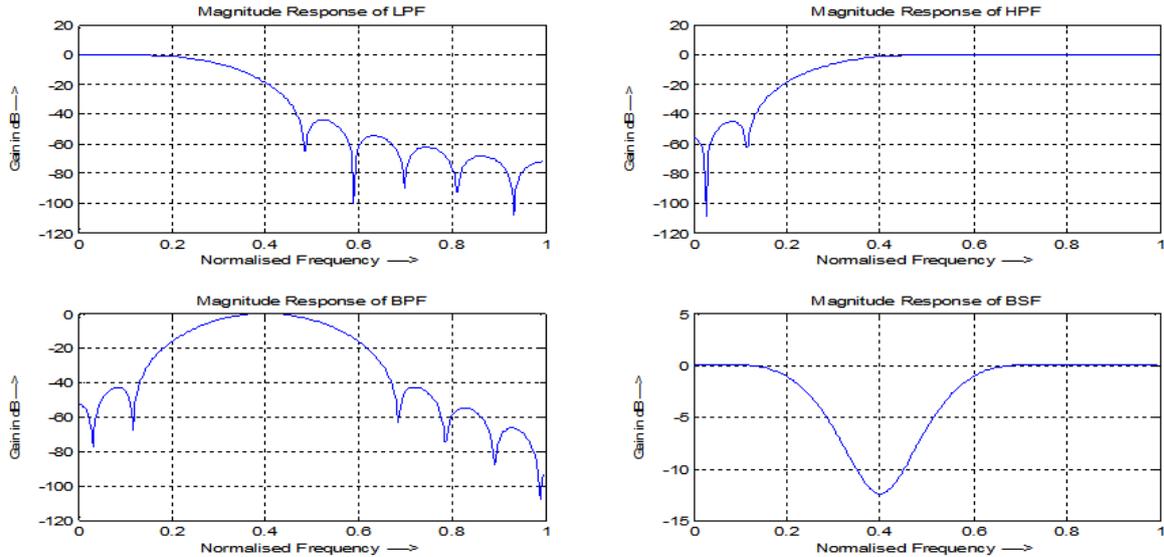


Figure.1 Hann window analysis

### 3.2. Hamming Window

Like Hann window the Hamming window is also one period of a raised cosine. However, the cosine is raised so high that its negative peaks are above zero, and the window has a discontinuity in amplitude leaving the window (stepping discontinuously from 0.08 to 0). This makes the side-lobe roll-off rate very slow. This modified cosine taper starts at 0.08, rises to 1 in the middle of the period, and then goes smoothly back down to 0.08 at the end.

The Hamming window is defined as

$$\omega(n) = \begin{cases} \alpha + \beta \cos \frac{2\pi n}{N-1}, & -\frac{N-1}{2} \leq n \leq (N-1)/2 \\ 0, & \text{otherwise} \end{cases}$$

Where  $\alpha, \beta$  are coefficients. For Hamming window the value of coefficients are

$$\alpha = 0.54 \text{ and } \beta = 1 - \alpha = 1 - 0.54 = 0.46.$$

Now the modified equation comes as,

$$\omega(n) = \begin{cases} 0.54 + 0.46 \cos \frac{2\pi n}{N-1}, & -\frac{N-1}{2} \leq n \leq (N-1)/2 \\ 0, & \text{otherwise} \end{cases}$$

Analysis is done for Hamming window for Lowpass filter, Highpass filter, Bandpass Filter and Bandstop filter.

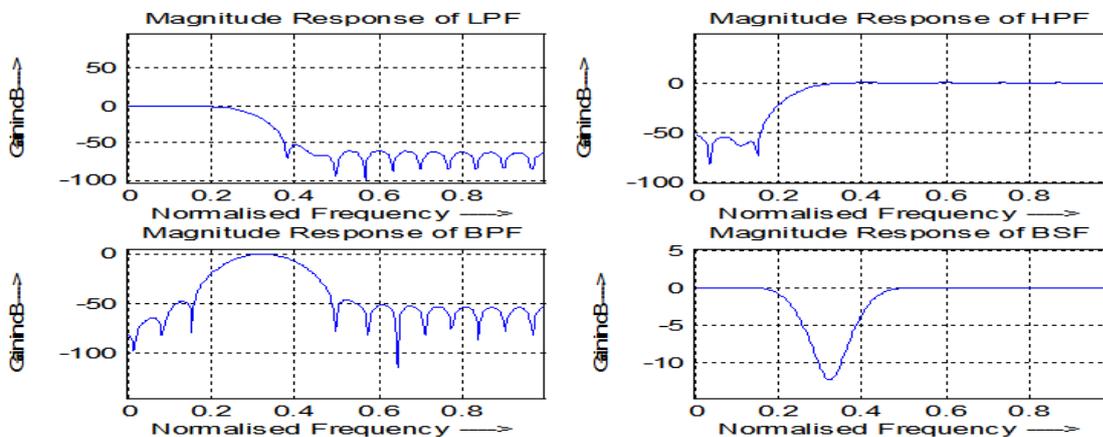


Figure.2 Hamming window analysis

### 3.3. Blackman Window

This modified cosine taper uses two cosines. The Blackman window is defined as

$$\omega(n) = \begin{cases} a_0 - a_1 \cos \frac{2\pi n}{N-1} + a_2 \cos \frac{4\pi n}{N-1}, & -\frac{N-1}{2} \leq n \leq (N-1)/2 \\ 0, & \text{otherwise} \end{cases}$$

Where  $a_0, a_1, a_2$  are coefficients. The coefficients can be explained with the help of

$$a_0 = \frac{1-\alpha}{2}; a_1 = 0.5; a_2 = \alpha/2$$

The value of  $\alpha$  for Blackman window is 0.16 and substituting this value in the expression for the window we get  $a_0 = 0.42; a_1 = 0.5; a_2 = 0.08$

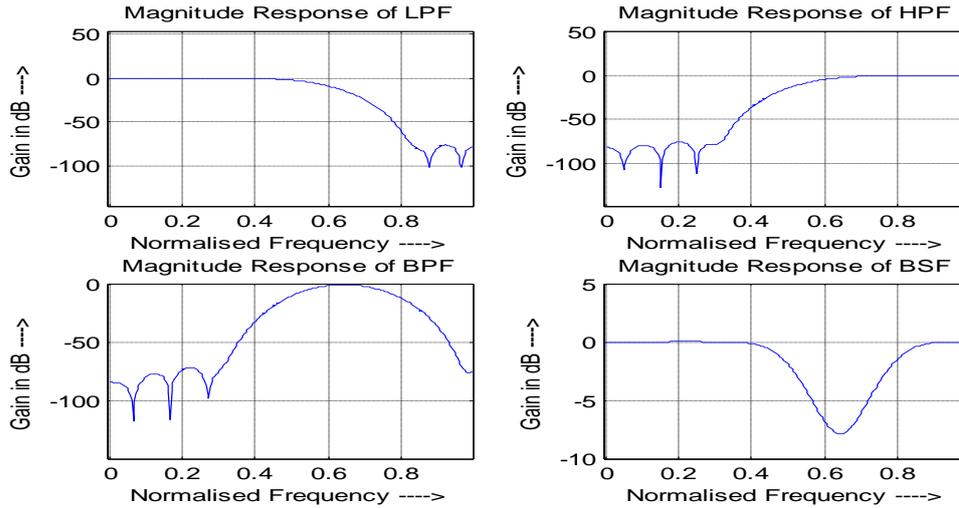


Figure.3 Blackman window analysis

### 3.4. Barlett Window

It is also called triangular window. Often applied discreetly to sample correlations of finite data. The expression for Barlett window is given by,

$$\omega(n) = \begin{cases} \frac{2n}{N-1}, & 0 \leq n \leq \frac{N}{2} - 1 \\ 2 - \frac{2n}{N-1}, & \frac{N}{2} \leq n < N \end{cases}$$

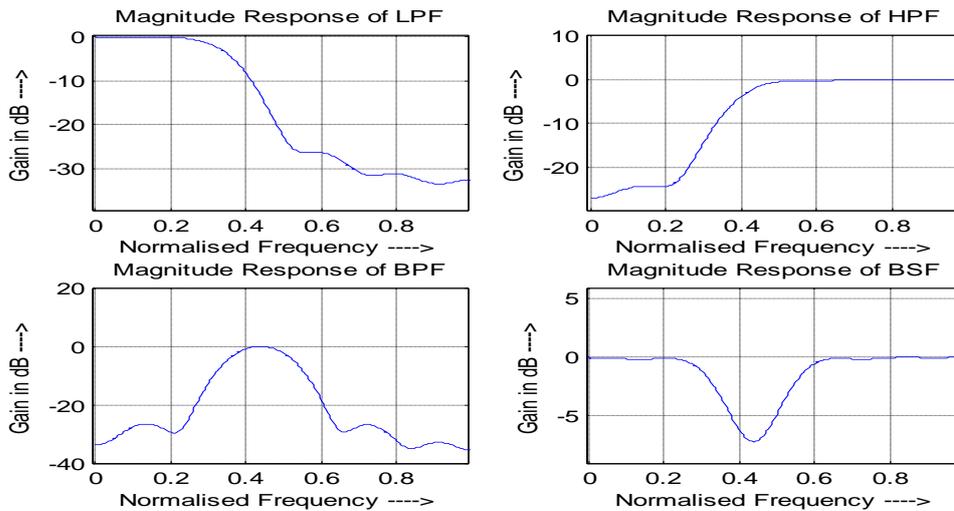


Figure.4 Barlett window analysis

#### IV. Analysis Of Simulation Results

In order to study window method different filters were considered. The major parameters which decides the quality of filters is the width of the main lobe , number of side lobes and the roll off. The desirable character is that width of main lobe should be high whereas the side lobes should be least and roll off should be high. For analysis Lowpass filter using Hann window, Hamming window , Blackman window and Barlett window are carried out.

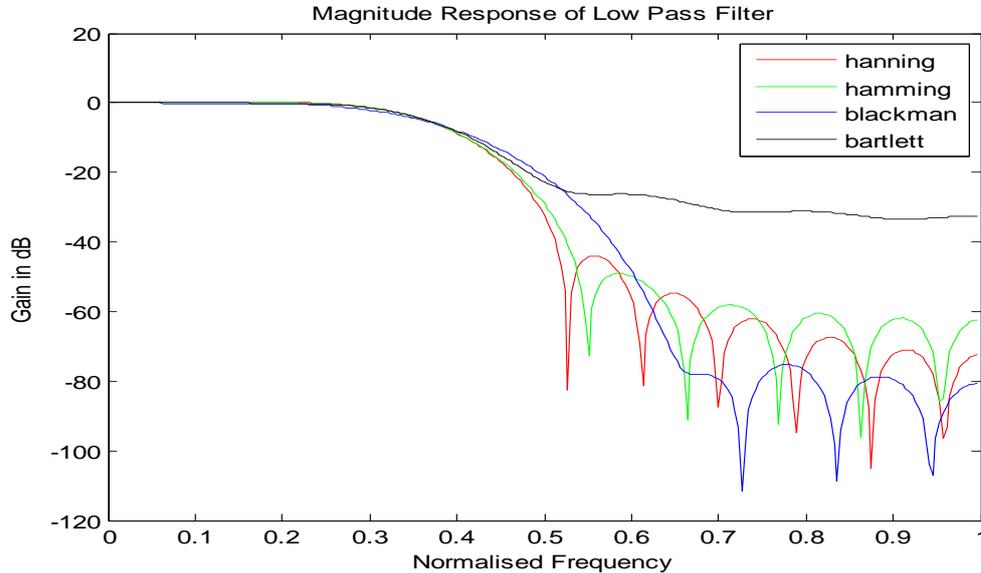


Figure.5 A comparative simulation of windws for Lowwpass filter

Table.1 Comparison between different window techniques

Window Method	Peak side lobe level(db)	Number of side lobes	First null(KHz)
Hann window	-39	5	1.65
Hamming window	-45	4	1.73
Blackman window	-70	3	2.08
Barlett window	-25	2	1.66

#### V. Conclusion

A study on different Finite Impulse Response windows were carried out. All the literature available on this subject helps us to select the different parameters required for comparing the varied windowing techniques[6][7][8][9].A comparative study of Barlett, Hann, Hamming and Blackman window was carried out using the FDA tool in MATLAB.Using the windows a Lowpass filter was simulated.Refer Fig.5.As it can be concluded that Hann window has the least main lobe width. For Hamming window the number of sidelobes are very high when compared with other all other windows. Future scope of the paper lies in the application of windows on a possible input signal say a speech signal and analyzing how effective they filter out the undesired components.

#### Acknowledgements

I would like to thank the management and Vice Chancellor of Rai Technology University to give me this opportunity to publish this paper.

#### References

##### Journal Papers:

- [1] Fredric J Harris, On the use of Windows for Harmonic Analysis with the Discrete Fourier Transform, Proceedings of the IEEE, Vol 66 ,No.1 , Jan 1978.
- [2] Sonika Gupta, Aman Panghal Performance Analysis of FIR Filter Design by Using Rectangular, Hanning and Hamming Windows Methods,IJARCSSE, Vol-2,Issue-6,Jun2012,pp273-278
- [3] Manira Khatun, Implement a new Window function and Design FIR filters by using this new window, IJECS,Volume-3,Issue-3, March 2014, pp4087-4090

- [4] Gopika P, Dr.Supriya Subash, Implementation of Space Time Adaptive Processing in Active SONAR detection,IOSR,Vol 9,Issue 3,May 2014 , pp 73-76.
- [5] Walt Kester ,Digital filters
- [6] HESC686 Mathematics and Signal Processing for Biomechanics
- [7] Window function , Wikipedia, the free encyclopedia,[https://en.wikipedia.org/wiki/Window\\_function](https://en.wikipedia.org/wiki/Window_function)

**Books:**

- [8] Sanjit K Mitra, James F. Kaiser, Handbook for Digital Signal Processing, John Wiley & Sons, Inc, 1993.
- [9] John G. Proakis and Dimitris G Manolakis, Introduction to Digital Signal Processing, MacMillian,1988.
- [10] Oppenheim , R Schafer, J.Buck, Discrete-Time Signal Processing, Second Edition , Prentice-Hall , 1999.